Charged lepton contributions to the solar neutrino mixing and θ_{13}

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Abstract

A charged lepton contribution to the solar neutrino mixing induces a contribution to θ_{13}, barring cancellations/correlations, which is independent of the model building options in the neutrino sector. We illustrate two robust arguments for that contribution to be within the expected sensitivity of high intensity neutrino beam experiments. We find that the case in which the neutrino sector gives rise to a maximal solar angle (the natural situation if the hierarchy is inverse) leads to a θ_{13} close to or exceeding the experimental bound depending on the precise values of θ_{12}, θ_{23}, an unknown phase and possible additional contributions. We finally discuss the possibility that the solar angle originates predominantly in the charged lepton sector. We find that the construction of a model of this sort is more complicated. We comment on a recent example of natural model of this type.

1 Introduction

The steady, remarkable progress we have witnessed in recent years in experimental neutrino physics has enabled a significant advance in our understanding of the lepton sector. The case for three neutrino oscillations is now compelling (although a full oscillation pattern still has to be observed) and the peculiar neutrino mass and mixing pattern observed represents a non-trivial handle on their origin. It is then natural to wonder what the understanding we gained implies for the value of the observables still to be measured, in particular θ_{13}. A major part of the rich neutrino experimental program available and partially under way will in fact focus on measuring that mixing angle, which has also important implications for leptonic CP violation and astrophysics. While many models do provide predictions for θ_{13}, the number of possibilities is high enough to make almost any value of θ_{13}, from zero to the present bound, compatible with some model. In this letter,

we will therefore try to overcome the bulk of the model dependence in those predictions by focusing on general mechanisms leading to calculable contributions to θ_{13}.

While both the neutral and charged lepton sectors contribute to lepton mixing, most of the uncertainties in model building come from the neutrino sector. This is because the light neutrino mass matrix is still less constrained than the charged lepton one and, more important, because the origin of its smallness introduces additional degrees of freedom. For example, in the case of the type I see-saw mechanism, the light neutrino matrix is determined by two independent mass matrices[1]. We will therefore concentrate on contributions to θ_{13} that are independent of the model building in the neutrino sector and rely instead on properties of the charged lepton sector[2]. In particular, we identify a contribution to θ_{13} induced by the charged lepton contribution to the solar mixing angle, which arises in the absence of correlations in the charged lepton mass matrix. We discuss two motivated expectations for the size of the latter contribution, both leading to values of θ_{13} likely to be within the reach of high intensity conventional neutrino beam experiments. In particular, we consider the case in which the neutrino sector gives rise to a maximal solar angle. This is typically the case in models with inverse hierarchical neutrinos [11]. In this case, the charged lepton sector must account for the observed deviation of θ_{12} from $45°$, which in turn leads to a contribution to θ_{13} close to the present experimental bound [11, 12]. We provide a simple analytical expression for the latter involving one physical phase and we plot the distribution of the corresponding numerical expectation taking into account the uncertainty on the mixing parameters and possible additional contributions [13]. Indeed, since additional contributions to θ_{13} may be present, accidentally cancelling the piece controlled by the charged lepton sector, the expectations we find should be considered as lower limits on θ_{13}, uncertain by a factor of order one. A condition for a non accidental cancellation is discussed in [14].

We also discuss the possibility that the charged lepton contribution to the solar mixing angle is large and accounts for most of it. We first consider the case in which the entries in the charged lepton mass matrix are uncorrelated and show that i) the induced contribution to θ_{13} is well beyond the experimental bound (the "θ_{13} tuning problem") and that ii) in SU(5) grand unified models, the electron mass (or the up quark mass) gets a contribution way larger than the observed value (the "m_e tuning problem", numerically more relevant). This makes the construction of a natural model of this sort considerably more complicated [13]. We then discuss the case in which correlations are present in the charged lepton mass matrix. In particular, we consider the possibility, recently pointed out in [14], that such correlations arise from a non-accidental physical mechanism. A condition on the charged lepton mass matrix sufficient to naturally solve the θ_{13} tuning problem has been identified in [14]. We show that an additional condition is necessary to ensure that the m_e tuning problem is also solved in SU(5) models. The mechanism illustrated in [14] may easily account for the latter condition as well.

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[1]One way of getting oriented in the jungle of model building possibilities in the neutrino sector is by considering minimal models [1]. See also the general arguments in [2, 3, 4, 5]. In [6] the assumption is made that the neutrino contribution to the mixing matrix is bimaximal. In some cases the model building in the neutrino and charged lepton sector can be closely related [7].

[2]This contribution arises in a number of explicit models, in some cases providing a precise prediction for θ_{13}, see e.g. [8, 3, 9, 10, 6]. We focus here on the general features of the effect.

2 Charged lepton rotations and θ_{13}

The neutrino mixing matrix U is the combination of two unitary matrices, U_e and U_ν, entering the diagonalization of the charged lepton and neutrino mass matrices m_E and m_ν[3],

$$U = U_e U_\nu^\dagger, \text{ where } m_E = U_{ec}^T m_E^{\text{diag}} U_e, \quad m_\nu = U_\nu^T m_\nu^{\text{diag}} U_\nu. \tag{1}$$

The decomposition of U into U_e and U_ν is of course not physical in the (minimally extended) Standard Model (SM)[4]. On the other hand, any attempt at investigating the origin of the fermion flavour structure involves physics beyond the SM that identifies a privileged basis in flavour space. This is the basis in which the entries of the fermion mass matrices are most simply related to the physics originating them and in which possible correlations among the entries in m_E, m_ν have to be considered accidental or related to symmetries or other physical mechanisms of the underlying theory.

We can then wonder if lepton mixing, and in particular the two large mixing angles observed in atmospheric, solar and terrestrial neutrino experiments, originate from the neutrino or the charged lepton sector. It is well known that the atmospheric angle θ_{23} can equally well originate from the neutrino [15] or the charged lepton [16] sector. The solar angle can in principle also come from both sectors. However, as we will see later on, cancellations or correlations in the charged lepton mass matrix are required in the case in which the origin of the solar angle is in the lepton sector. For the time being, we consider the case in which the entries of m_E are not correlated and show that in general a left-handed 12-rotation in the charged lepton sector θ_{12}^e induces a contribution to θ_{13}. This derivation is closely related to that in [3].

Barring correlations in the charged lepton mass matrix m_E[5], the hierarchy of charged lepton masses translates in a hierarchical structure of m_E:

$$\begin{vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{vmatrix}^E \ll \begin{vmatrix} m_{22} & m_{23} \\ m_{32} & m_{33} \end{vmatrix}^E m_\tau \ll |m_{33}|^E m_\tau^2. \tag{2}$$

Note that the condition (2) is compatible with asymmetrical matrices and in particular with the atmospheric angle originating from m_E.

The charged lepton mass matrix can be approximately diagonalized by the subsequent diagonalization of his 2×2 blocks. As a consequence of (2), the diagonalization of the heaviest 23 block should be performed first, so that

$$U_e = U_{12}^e U_{13}^e U_{23}^e, \tag{3}$$

where U_{ij}^e is a complex rotation in the ij block. The ordering (3) ensures that the mixing parameters are directly related to the entries of m_E and are therefore independent, barring correlations already present in m_E or induced by the first steps of diagonalization. For our purposes, it is sufficient to consider the two cases $U_{12}^e \neq \mathbf{1}$, $U_{13}^e = \mathbf{1}$ and $U_{13}^e \neq \mathbf{1}$, $U_{12}^e = \mathbf{1}$.

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[3] We use a convention in which the left-handed fields lie on the right hand side of m_E.

[4] An $SU(2)_L$-invariant transformation in flavour space can equally well rotate the physical mixing in the neutrino and in the charged lepton sector.

[5] Democratic models are an example in which such correlations are not accidental.

Since the two possibilities are actually equivalent (up to a relabelling of the first two rows of m_E) in the $\theta_{23} = \pi/4$ limit, in this paper we will consider the $U_{12}^e \neq \mathbf{1}$, $U_{13}^e = \mathbf{1}$ case only.

When combining U_e and U_ν in the physical neutrino mixing matrix, the U_{12}^e rotation ends up on the left-hand side of U:

$$U = U_{12}^e \hat{U}, \tag{4}$$

where $\hat{U} = U_{23}^e U_\nu^\dagger$ and the standard parameterization can be used for U_ν. Note that, wherever it comes from, the atmospheric angle resides in $\hat{U}$. Now, in order to read the value of the solar angle θ_{12} from eq. (4) we have to write U in the parameterization that defines θ_{12}, in which the 12 rotation lies on the right-hand side: $U = U_{23}U_{13}U_{12}$. This means that U_{12}^e has to be commuted with $\hat{U}$ and in particular with the large 23 rotation, thus inducing a contribution $\hat{\theta}_{13}$ to θ_{13} which is easily found to be given by

$$\sin\hat{\theta}_{13} = \sin\theta_{12}^e \frac{\tan\theta_{23}}{\sqrt{\cos^2\theta_{12}^e + \tan^2\theta_{23}}} \simeq \sin\theta_{12}^e \sin\theta_{23} \ , \tag{5}$$

where θ_{12}^e is the angle associated to U_{12}^e and the approximated expression holds for small θ_{12}^e. Eq. (5) generalizes results in [3, 17, 10]. By construction, under the present assumptions the contribution in eq. (5) is independent of possible additional contributions to θ_{13}. A cancellation among them would be accidental.

The contribution to θ_{13} we obtain this way does not depend on the many unknowns associated with the model building in the neutrino sector. In particular, it is independent of the form of the light neutrino mass matrix; of the mechanism accounting for its smallness; of the form of the Majorana and Dirac mass matrices in see-saw models. It is also independent of the origin of the atmospheric mixing angle (neutrino or charged lepton sector).

On the other hand, $\hat{\theta}_{13}$ does depend on properties of the charged fermion sector, such as the size of the θ_{12}^e rotation and the independence of the entries of m_E. In the next section we illustrate two arguments on the expected size of θ_{12}^e.

3   Two arguments on the size of θ_{12}^e

We first consider a well known, robust ansatz on the structure of the light 2×2 block of the charged fermion mass matrices. The ansatz is characterized by a negligible 11 element and by the approximate equality of the absolute values of the 12 and 21 elements [18, 9]. Such a pattern can be accounted for by an elegant non-abelian symmetry [19]. More important, it leads to the successful and precise (at the 5% level) relation $|V_{us}| = \sqrt{m_d/m_s}$. Furthermore, in a SU(5) grand unified model, the relations $m_{12(21)}^E = m_{21(12)}^D$, together with the Georgi-Jarlskog factor 3 in $m_{22}^E = 3m_{22}^D$ (necessary to account for the muon mass), lead to a second successful prediction: $m_e/m_\mu = (m_d/m_s)/9$. A prediction for θ_{12}^e also follows, which can certainly be considered well motivated, in the light of the above. The prediction is $\theta_{12}^e = (m_e/m_\mu)^{1/2} \simeq 0.07$ [8, 9], leading to

$$\sin\hat{\theta}_{13} \simeq \sqrt{m_e/m_\mu} \sin\theta_{23} \simeq 0.05 \ , \tag{6}$$

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3–4 times below the present experimental value and therefore within the reach of future high intensity neutrino beam experiments.

In the presence of additional contributions, eq. (6) should be interpreted as a lower bound on θ_{13} that can be evaded if a cancellation occurs. In order to estimate how small θ_{13} can be made by such a cancellation, we add to $\sin\hat{\theta}_{13}$ a random contribution larger than 10^{-4}, with flat logarithmic distribution and arbitrary phase. We then obtain a probability distribution for θ_{13}, which is shown in Figure 1a. The peak at $\theta_{13} \simeq \hat{\theta}_{13}$ is due to the small values of the additional contribution (which all give $\theta_{13} \simeq \hat{\theta}_{13}$) and is therefore not particularly meaningful. More meaningful is the range of θ_{13} in which the probability distribution is not too much suppressed with respect to the plateau on the right of the peak. We find that the suppression factor is larger than 5 for $\theta_{13} < 0.02$. In other words, a cancellation leading to $\theta_{13} < 0.02$ is unlikely. Figure 1a is a (mild) generalization of Fig. 1b in [9], where the additional contribution comes from a specific structure of the neutrino mass matrix. See also [3, 10].

The case in which the neutrino mass spectrum is of the inverted type provides a different, purely phenomenological constraint on θ_{12}^e. Strictly speaking, the argument follows from assuming that the matrix $\hat{U}$ in eq. (4) is in the form $\hat{U} = U_{23}(\theta)U_{12}(\theta_{12} = \pi/4)$. This is the typical situation in models leading to the inverted neutrino mass pattern. In such models, in fact, the deviation of the neutrino contribution to the solar angle from $\pi/4$ is related to $\Delta m_{21}^2/\Delta m_{32}^2$ and turns out to be small compared to the observed deviation [11], barring tunings[6]. Depending on the model, the atmospheric angle can come from the neutral or the charged lepton sector, but in both cases $\hat{U}$ has the above form, up to a possible further 13 rotation, which we neglect. A sizable θ_{12}^e is required in this case to account for the observed value of θ_{12}, significantly different from $\pi/4$ [11, 12]. The relation between the deviation of θ_{12} from $\pi/4$ and the required θ_{12}^e involves a physical phase ϕ [1, 13]:

$$\begin{cases} \sin\theta_{13} = \sin\theta\sin\theta_{12}^e \\ \tan\theta_{23} = \cos\theta_{12}^e\tan\theta \\ \tan\theta_{12} = \left|\dfrac{1 - \cos\theta\tan\theta_{12}^e e^{i\phi}}{1 + \cos\theta\tan\theta_{12}^e e^{i\phi}}\right| \end{cases}. \tag{7}$$

Solving for θ_{12}^e, we obtain the following equation for $\sin\theta_{12}^e/\sqrt{\cos^2\theta_{12}^e + \tan^2\theta_{23}} \equiv x = \sin\theta_{13}/\tan\theta_{23}$:

$$\frac{2x}{1+x^2} = \frac{\cos 2\theta_{12}}{\cos\phi} \tag{8}$$
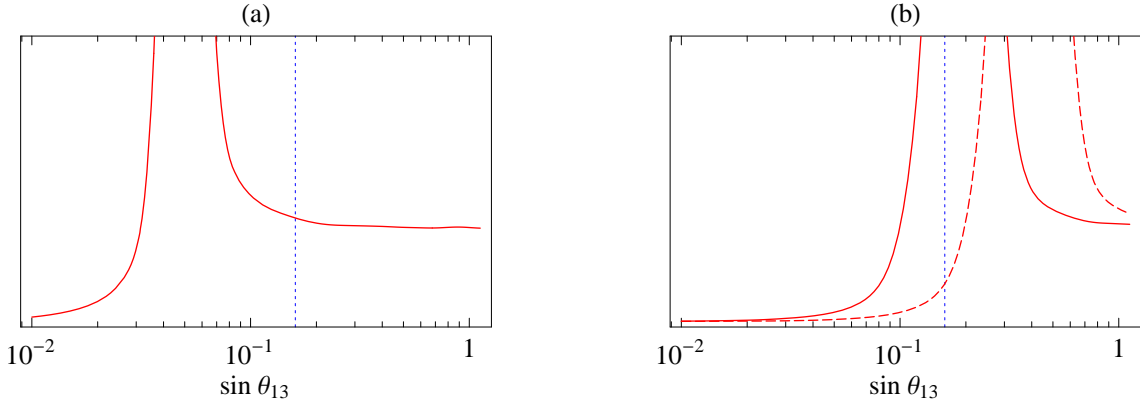
or, neglecting terms quadratic in x,

$$\sin\theta_{12}^e = \frac{\cos 2\theta_{12}}{2\cos\theta_{23}\cos\phi}. \tag{9}$$

At the same order, θ_{13} is given by

$$\sin\theta_{13} = \frac{\tan\theta_{23}}{2}\frac{\cos 2\theta_{12}}{\cos\phi}. \tag{10}$$

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[6]If the atmospheric angle comes from the charged lepton sector, the 11 and 22 elements of the neutrino mass matrix should to be tuned to be equal up to a phase. This could be obtained in a non accidental way by using a non abelian symmetry.

(a)   (b)

10^{-2} 10^{-1} 1 10^{-2} 10^{-1} 1
          $\sin\theta_{13}$ $\sin\theta_{13}$

Figure 1: Distribution of θ_{13} obtained by adding a random contribution to $\sin\hat{\theta}_{13} = \sqrt{m_e/m_\mu}\sin\theta_{23}$ (a) or $\sin\hat{\theta}_{13} = \tan\theta_{23}/2\cos2\theta_{12}/\cos\phi$ (b). The vertical dotted line represents the present limit. The case $\cos\phi = 1$ (solid) and $\cos\phi = 0.5$ (dashed) are both shown in (b). See comments in the text.

In explicit models, the phase ϕ can be related to the leptonic CP-violating phase in the standard parameterization [1]. Numerically, the prediction for θ_{13} is very close to or beyond the experimental bound, depending on $\cos\phi$ and on the values used for θ_{12}, θ_{23}. In Figure 1b we show the distribution for $\sin\theta_{13}$ we obtain by using the present distributions for θ_{23} and θ_{12} and by adding a random contribution as before. The solid line corresponds to $\cos\phi = 1$, whereas the dashed line corresponds to $\cos\phi = 0.5$. In the case $\cos\phi = 1$, the probability is suppressed by a factor of more than 5 (compared with the plateau on the right of the peak) when $\theta_{13} < 0.07$. When $\cos\phi < 0.5$, this happens in the whole allowed range.

Note that eq. (9) is not compatible with the powerful ansatz described at the beginning of this section, since it requires a larger θ_{12}^e ($\theta_{12}^e \sim 0.25$–0.30). In order to avoid a large contribution to the electron mass, this in turn requires a significant asymmetry in the 12 block of m_E.

4 θ_{12} from the charged lepton sector (no correlations in m_E)

Let us now consider the possibility that the solar angle originates predominantly in the charged lepton sector. We first consider again the case in which the entries in m_E are independent. Then θ_{12} must originate from the U_{12}^e factor in (4). Since the required size of the θ_{12}^e rotation is now quite large,

$$\sin\theta_{12}^e = \frac{\sin\theta_{12}}{\cos\theta_{23}} \simeq 0.8 \, , \tag{11}$$

we can expect the induced contribution to θ_{13} to be large. Indeed, in the limit in which the matrix $\hat{U}$ in (4) consists of a pure 23 rotation, we have

$$\sin\hat{\theta}_{13} = \tan\theta_{23}\tan\theta_{12} \simeq 0.6\text{–}0.7, \tag{12}$$

4–5 times above the experimental limit. Therefore, the matrix $\hat{U}$ should contain an additional 13 or 12 rotation factor cancelling most of $\hat{\theta}_{13}$. We will refer to this problem as

the "θ_{13} tuning problem". In this scenario, one of the few hints on the origin of lepton mixing (the smallness of θ_{13}) would be an accident. On the other hand, the degree of cancellation needed to bring θ_{13} below the experimental limit is mild. A potentially more serious tuning problem comes from the $m_e \ll m_\mu$ hierarchy, at least in SU(5) models, as we now see.

Let us call $\theta_{12}^{e_c}$ the right-handed rotation involved in the diagonalization of the light 2×2 block of the charged lepton matrix (after diagonalization of the 23 block). The e/μ mass ratio is then given by

$$\frac{m_e}{m_\mu} \simeq \left| \frac{m_{11}^E}{\hat{m}_{22}^E} - \tan\theta_{12}^e \tan\theta_{12}^{e_c} \right| , \tag{13}$$

where $\hat{m}_{22}^E$ is the 22 entry of m_E after diagonalization of the 23 block. As a consequence, we expect m_e/m_μ to be of the order of $\tan\theta_{12}^e \tan\theta_{12}^{e_c} \sim \tan\theta_{12}^{e_c}$ or larger, barring cancellations. On the other hand, $\theta_{12}^{e_c}$ is related to the left-handed 12 rotation in the down quark sector θ_{12}^d (analogous of θ_{12}^e) by GUT relations: $\theta_{12}^d = C\theta_{12}^{e_c}$, where C comes from possible SU(5) Clebsch-Gordan factors. We also know that θ_{12}^d contributes to the CKM element V_{us}. Indeed, if the up quark mass is to be naturally small, the up quark contribution to V_{us} must be subdominant. That is because the up quark matrix is symmetrical in SU(5) and $\theta_{12}^u \sim |V_{us}|$ would give a large contribution to m_u/m_c through a relation similar to (13). We then have $\theta_{12}^d \simeq |V_{us}|$ and a contribution to m_e/m_μ of order $|V_{us}|/C$. In minimal SU(5), this would be about 50 times larger than the measured value. Even in the presence of a (plausible) Clebsch-Gordan factor, the necessary tuning is still larger than that involved in the θ_{13} tuning problem. We refer to this problem as the "m_e tuning problem".

In summary, in absence of correlations in m_E, generating the solar mixing angle from the charged lepton sector requires cancellations in the determination of θ_{13} and, in SU(5) models, in the determination of the electron (or up quark) mass.

5 θ_{12} from correlations in m_E

Let us now consider the case in which the entries of the charged lepton mass matrix are correlated. It has recently been shown [14] that such correlations can be naturally induced in a Froggat–Nielsen context by the dominance of a heavy vector-like lepton exchange (analogous to the single right-handed neutrino dominance scenario in the neutrino sector [15]). More precisely, the approximate vanishing of the determinant of the x_{ij} coefficients in

$$m_E \propto \begin{pmatrix} a\epsilon' & b\epsilon' & \mathcal{O}(\epsilon') \\ x_{21}\epsilon & x_{22}\epsilon & \mathcal{O}(\epsilon) \\ x_{31} & x_{32} & 1 \end{pmatrix} \tag{14}$$

($\epsilon' \ll \epsilon \ll 1 \sim a, b, x_{ij}$) has been identified as a condition for the large θ_{12} (and θ_{23}) to originate in a natural way from m_E. The correlations in m_E translate in fact in a relation between the 12, 13, 23 rotations in eq. (3) cancelling the physical θ_{13} angle. This can easily be seen by observing that, unlike the case in which correlations are absent, the proper way to diagonalize m_E is by performing the left-handed 12 rotation first, followed by the 23 rotation.

The form of m_E in (14) is not unique. For example, the pattern

$$\begin{pmatrix} \epsilon' & 0 & x_{13}\epsilon \\ 0 & \epsilon' & x_{23}\epsilon \\ x_{31} & x_{32} & 1 \end{pmatrix} , \tag{15}$$

with $|x_{13}x_{32} - x_{31}x_{23}| \ll 1$, also gives a large solar angle in a natural way. However, it is not compatible with SU(5) (see below) and it is harder to obtain from the model building point of view (see however [7]). Another possibility is that correlations are not present in the initial form of m_E but they are induced by the first steps of the diagonalization in (3). In the following we will concentrate on the possibility in eq. (14).

While the $|x_{21}x_{32} - x_{31}x_{22}| \ll 1$ condition on the coefficients in eq. (14) does solve the θ_{13} problem discussed in the previous section, the additional condition

$$|ax_{22} - bx_{21}| \ll 1 \tag{16}$$

must be imposed in order to ensure that the electron mass and the up quark mass are naturally small in SU(5) grand unified models (namely to ensure that the m_e problem illustrated above is also solved). The only possibility to escape the condition (16) is boosting the 13 entry in (14), namely imposing the alternative condition $|m_{11}^E|$, $|m_{12}^E| \ll |m_{13}^E|$.

The argument goes as follows. In SU(5) the light block of the up quark mass matrix is symmetric. As a consequence, the up quark contribution to $|V_{us}|$ must be subdominant, as discussed in the previous Section, and we can identify $|V_{us}|$ with its down quark contribution. We then obtain, e.g. in minimal SU(5),

$$\frac{m_e}{m_\mu} \cos \theta_{23} \sim \left| V_{us} \frac{ax_{22} - bx_{21}}{ax_{21} + bx_{22}} \right| . \tag{17}$$

The condition (16) follows from $|V_{us}| \gg m_e/m_\mu \cos \theta_{23}$. We have used $\theta_{12}^{e_c} \simeq |V_{us}|$ and we have neglected the contribution of the 13 entry in (14) to $\theta_{12}^{e_c}$ that arises due to the 23 rotation. If that entry is larger than $\mathcal{O}(\epsilon')$, its contribution to $\theta_{12}^{e_c}$ and V_{us} dominates and the connection with m_e/m_μ is lost, thus leading to the alternative possibility $|m_{11}^E|$, $|m_{12}^E| \ll |m_{13}^E|$.

The above can be rephrased by using the example in Section 1 of [14]. Assume that the lepton mixing matrix is in the form $U = U_{23}(\theta_{23} = \pi/4)U_{12}(\theta)$. Then, using the notation of eq. 4 in [14],

$$m_E = V_e \begin{pmatrix} c\,m_e & s\,m_e & 0 \\ s/\sqrt{2}\,m_\mu & -c/\sqrt{2}\,m_\mu & m_\mu/\sqrt{2} \\ -s/\sqrt{2}\,m_\tau & c/\sqrt{2}\,m_\tau & m_\tau/\sqrt{2} \end{pmatrix} . \tag{18}$$

Given the relation between $(V_e)_{12}$ and V_{us}, taking $V_e \sim 1$ is not a good approximation in SU(5). For instance, in minimal SU(5) we have

$$m_E \simeq \begin{pmatrix} V_{us}s/\sqrt{2}m_\mu & -V_{us}c/\sqrt{2}m_\mu & V_{us}m_\mu/\sqrt{2} \\ s/\sqrt{2}m_\mu & -c/\sqrt{2}m_\mu & m_\mu/\sqrt{2} \\ -s/\sqrt{2}m_\tau & c/\sqrt{2}m_\tau & m_\tau/\sqrt{2} \end{pmatrix} . \tag{19}$$

Additional contributions to the first and second rows proportional to V_{ub} and V_{cb} might also be relevant, but they have been omitted in (19).

From the model building point of view, the additional condition (16) does not represent an additional challenge. The same mechanism accounting for the correlation in the lower left block of m_E may well account also for the correlation in the upper left block. This is illustrated by the explicit SU(5) model in Appendix A of [14]. In the light of eq. (19), we remark however that one might expect the first row of m_E to be suppressed by only a factor $|V_{us}|$ compared to the second row.

6 Summary

We have shown that a charged lepton contribution to the solar neutrino mixing induces a contribution to θ_{13}, barring cancellations/correlations, which is independent of the model building options in the neutrino sector. We have illustrated two robust arguments for that contribution to be within the expected sensitivity of high intensity conventional neutrino beam experiments. The corresponding expectations for θ_{13} are shown in Figure 1. In particular, in the case in which the neutrino sector gives rise to a maximal solar angle (the natural situation if the neutrino spectrum is inverted, barring non accidental tunings) we have given a simple analytical expression for the induced contribution to θ_{13} taking into account the dependence on a physical phase. The numerical value of θ_{13} turns out to be very close or exceeding the experimental bound, depending on the precise value of the solar and atmospheric mixing angles, the value of the phase, and the presence of additional contributions. We have also discussed the possibility that the solar angle originates predominantly in the charged lepton sector. In the case in which no correlations are forced in the charged lepton mass matrix, we have shown that this possibility faces two fine-tuning problems, one in the determination of θ_{13} and one in the determination of the electron (or up quark) mass in SU(5) models. We have also considered the case in which correlations are present in the charged lepton mass matrix and, in the context of the possibility discussed in [14], we have identified the additional conditions that allow to solve the m_e tuning problem in SU(5) models.

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